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An approximate thermal analysis for a regenerative heat exchanger

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Abstract—This paper describes an approximate thermal analysis of a regenerative heat exchanger. The approximation used relies on the fact that the dimensionless parameters, known as the reduced periods, are not too large, a condition which is made more precise in the paper, and which is true for all power station air heaters. There is no upper limit on the reduced lengths with which the method can cope. The method gives time averaged fluid outlet temperatures which are equivalent to those deduced from an analysis of an equivalent recuperator by Hausen (*Verfahrenstecknik Z. Ver. Dt. Ing.* **2,** 31–43 (1942)). The method also predicts the variation of fluid and heat exchange element temperatures with position and time. The method is generalized to cover the case of a regenerator with two or more zones with different heat transfer coefficients in each zone. This has applications in power station regenerative air heaters where different heat exchange elements are frequently used in the hot and cold zones of the heater, and to high-temperature regenerators where the variation of fluid properties with temperature can be approximated by splitting the regenerator into a number of zones, with constant properties in each zone. The results are compared with a finite difference solution of the regenerator problem for sets of plant data. The ability of the method to cope with long regenerators is also demonstrated, and the results are compared with Hausen's solution and with standard results obtained from the literature.

1. INTRODUCTION

REGENERATIVE heat exchangers are widely used in industrial processes for the exchange of heat between two gas streams. A regenerator consists of a porous packing of solid material through which fluids can flow. In the operation of a regenerator, a hot gas (e.g. flue gas leaving a power station boiler) is passed through the solid, and gives up heat to the solid. Subsequently a cold fluid (e.g. combustion air for the boiler) is passed through the solid in the opposite direction, and receives heat from the solid. In a fixed bed system at least two regenerators are required for continuous operation, and the periodic reversal of flow in each regenerator is achieved by a series of valves. An alternative configuration is the rotary regenerator in which the solid heat exchange material is packed in a drum. This drum is then rotated relative to the air and gas streams by either rotating the drum, or by using a stationary drum and directing the flow through rotating headers (Fig. 1).

Regenerators have advantages over recuperators (heat exchangers in which the hot and cold fluids flow continuously and exchange heat via a common solid wall) in a number of applications. In high-temperature applications they allow the use of simple materials for the heat exchange material, such as firebricks or ceramic checkers. The classic high-temperature application of a regenerator is the Cowper Stove as used in the steel industry. A second advantage of regenerators occurs at more modest temperatures where dirty or corrosive gases are used. It is straightforward to allow for cleaning of the heat transfer surface of a regenerator, e.g. by high-pressure air or steam jets, while maintaining a compact design of heat exchanger.

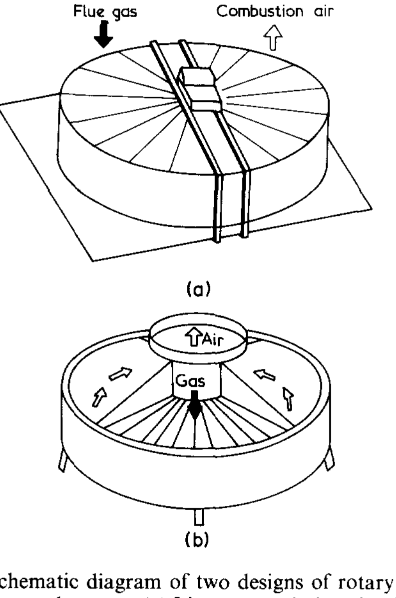

FIG. 1. Schematic diagram of two designs of rotary regenerative heat exchangers: (a) Ljungstrom design, fixed ducts, rotating drum; (b) Rothemuhle design, stationary drum, rotating headers.

From the above, it is clear that a regenerator always operates in a transient mode, in contrast with a recuperator, for which a genuine steady state is possible. Under steady operating conditions a regenerator will achieve a quasi-steady state in which the temperature oscillations are the same from one cycle to

NOMENCLATURE

A	time averaged air temperature
a	amplitude of air temperature oscillation
B	constant
b	constant
C	heat capacity of metal per unit volume
c_p	specific heat of fluid
E	$\exp(f)$
$\mathbf{F}$	vector of fundamental solutions
f	exponent in recuperator solution
G	time averaged gas temperature
g	amplitude of gas temperature oscillation
H	metal surface area per unit volume
h	heat transfer coefficient
k	ratio of gas and air reduced lengths
l	length of channel
M	time averaged metal temperature during gas cycle
m	amplitude of metal temperature oscillation during gas cycle
N	time averaged metal temperature during air cycle
n	amplitude of metal temperature oscillation during air cycle
T	temperature
t	time
t_a	period of air cycle
t_g	period of gas cycle
u	dimensionless temperature
W	mass flow of fluid per unit approach area
x	length
y	dimensionless length
z	position of change in element type.

Greek symbols

α	exponent in general solution
β	parameter describing degree of unbalance
ε	small parameter
Λ	reduced length
μ	coefficient of quadratic term in metal temperature during gas cycle
ν	coefficient of quadratic term in metal temperature during air cycle
π	reduced period
τ	dimensionless time
χ	weight function for the heat balance integral method.

Subscripts

a	air cycle
c	coldest value reached during a complete cycle
g	gas cycle
h	hottest value reached during a complete cycle
i	inlet
j	index denoting fundamental solutions
m	metal
n	index denoting region
o	outlet.

the next. The thermal performance of the regenerator can be described in terms of the time averaged outlet temperatures of the air and gas streams. This problem has been considered by Hausen [1], who showed that the fluid outlet temperatures can be determined by considering an equivalent recuperator. Hausen's analysis showed how the heat transfer coefficients in the recuperator can be related to those in the regenerator in order that the fluid outlet temperatures are the same.

For other aspects of regenerator performance it is necessary to consider the solution in more detail. For a fixed bed regenerator the temperature swings at outlet from the regenerator can be important, while for a rotary regenerator the fluid temperature swings can lead to stratification in the fluid outlet ducts. The amount by which the heat exchange drum of a rotary regenerator will distort depends on the variation of the solid temperature through the regenerator. A knowledge of the thermal distortion is necessary to provide effective sealing between the stationary and moving parts of the regenerator. The corrosion and fouling of any section of the heat exchange elements will depend on the variation of the local fluid and solid temperatures [2].

In the following sections a simple approximate analytic method is developed for the solution of the quasi-steady state problem under the condition that dimensionless parameters known as the reduced periods are not much larger than two. This criterion is made more precise in the subsequent work. The method also requires the reduced length of the regenerator to be at least as large as the reduced period, however, there is no upper limit on the reduced length. These restrictions are shown to be satisfied for a selection of power station rotary generators. The solution gives the time averaged fluid and solid matrix temperatures at any point in the regenerator, and also predicts the fluid and solid temperature oscillations throughout the depth of the regenerator.

2. THE REGENERATOR EQUATIONS

A counterflow regenerative heat exchanger is considered under quasi-steady operating conditions. The fluids which are hot and cold at the inlet will be referred to as gas and air, respectively, their temperatures and physical properties are referred to with subscripts g and a, respectively. During the gas phase $(-t_g < t < 0)$, the fluid and metal temperatures (which are denoted by a subscript m) satisfy the equations [3]

$$\partial T_g/\partial x = -(h_g H/W_g c_{pg})(T_g - T_m) \quad (1)$$

and

$$\partial T_m/\partial t = (h_g H/C)(T_g - T_m) \quad (2)$$

where x denotes the distance from the gas inlet, T the temperature (with T_m the matrix temperature) and h, H, W, c_p and C the heat transfer coefficient, the metal surface area per unit matrix volume, the mass flow of fluid per unit approach area, the specific heat of the fluid and the heat capacity of the metal per unit matrix volume.

Equations (1) and (2) ignore the effects of heat conduction in the elements parallel to the direction of the fluid flow, the heat capacity of the fluid resident in the regenerator, and the difference between the average temperature and the surface temperature of the elements, all of which can be shown to be very small for a power station air heater.

Equations (1) and (2) can readily be made dimensionless by setting

$$u = (T - T_{ai})/(T_{gi} - T_{ai}) \quad (3)$$

where T_{gi} and T_{ai} are the gas and air inlet temperatures, and setting $y = x/l$ where l is the length of the channel and $\tau = t/t_g$ to give

$$\partial u_g/\partial y = -\Lambda_g(u_g - u_m) \quad (4)$$

$$\partial u_m/\partial \tau = \pi_g(u_g - u_m) \quad (5)$$

where the dimensionless constants $\Lambda_g = (h_g Hl/W_g c_{pg})$ and $\pi_g = (h_g H t_g/C)$ are exactly equivalent to the constants Λ and π used by Hausen. These constants are generally referred to as reduced length and reduced period, respectively. Equations (4) and (5) apply for $0 < y < 1$ and $-1 < \tau < 0$. In a similar way one can obtain dimensionless equations for the air phase. For these equations $y = x/l$ and $\tau = t/t_a$ are set to give

$$\partial u_a/\partial y = \Lambda_a(u_a - u_m) \quad (6)$$

$$\partial u_m/\partial \tau = \pi_a(u_a - u_m) \quad (7)$$

for $0 < y < 1$, $0 < \tau < 1$.

The appropriate boundary conditions are

$$u_g = 1 \quad \text{at } y = 0 \text{ for } -1 < \tau < 0 \quad (8)$$

and

$$u_a = 0 \quad \text{at } y = 1 \text{ for } 0 < \tau < 1. \quad (9)$$

Finally, conditions are required to describe the quasi-steady, periodic operation of the regenerator. These are

u_m continuous across $\tau = 0$

and for $0 < y < 1$ (10)

$u_m(\tau = -1) = u_m(\tau = 1)$.

To date two types of method have been developed to solve this problem. The problem can be solved numerically as an initial value problem and the solution continued until the periodicity conditions (equations (10)) are approximately satisfied [4]. Alternatively, Iliffe [5] and Nahavandi and Weinstein [6] solved the steady, periodic problem in terms of the solution to an integral equation.

The steady periodic problem has also been solved by Hill and Willmott [7] who used the method of lines to reduce the problem to a system of typically 20 or 30 ordinary differential equations in time. In Section 3 a simpler method is developed for the solution by approximating the time variation of temperature at any point as a polynomial. This method applies only to regenerators with a reduced period of $\pi = 2$ or less, a condition which is satisfied by the rotary air preheaters installed in power station boilers. However, there are no difficulties with the method if the reduced length of the regenerator is large, a condition which does lead to difficulties with some of the above methods.

3. FORMULATION OF THE METHOD

A technique which can be applied to obtain approximate analytic solutions to heat conduction problems is the heat balance integral method [8, 9]. A typical heat conduction problem may consist of a differential equation

$$L(u) = 0 \quad (11)$$

which applies for $0 < y < 1$, $\tau > 0$ together with appropriate boundary conditions. A function v is accepted as an approximate solution of the heat conduction problem if it satisfies the boundary conditions together with a weaker form of equation (11), namely

$$\int_0^1 L(v)\chi_j(y)\,dy = 0 \quad (12)$$

for some sequence of functions

$$\chi_j;\ j = 1, 2, \ldots, J. \quad (13)$$

The method provides no mechanism for estimating the accuracy of an approximate solution other than by comparing it with an exact or numerical solution to the problem. However, providing care is taken in choosing the form of the approximate solution, the method provides useful results.

A similar technique will be applied to obtain an approximate solution to the regenerator problem, except that for this problem it is convenient to integrate the differential equations with respect to time rather than distance. Hence instead of solving equation (4) an approximate solution will be accepted which satisfies

$$\int_{-1}^{0} (\partial u_g/\partial y + \Lambda_g(u_g - u_m))\chi_j(\tau)\,d\tau = 0 \quad (14)$$

for $j = 1, 2, \ldots, J$. The weak or integrated forms of equations (5)–(7) follow in a similar way. The approximate solution will be chosen so that the temperature distributions are polynomials in τ at any point along the length of the regenerator, i.e. the temperature is written as polynomials in the dimensionless time τ, the coefficients of which are functions of y. Because of the time derivative in equations (5) and (7) the polynomials for u_m are required to be one degree higher than those for u_g and u_a. If attention is restricted to cases where the reduced periods π_g and π_a are not too large and the ratios π/Λ are not larger than 1, the exact variation of temperature with time will have a 'sawtooth' pattern which can be reasonably well approximated by low-order polynomials. If u_m is approximated by a quadratic function of τ in each period and u_g and u_a by linear functions of τ, one can write

$$u_g \cong G(y)+g(y)(\tau+\tfrac{1}{2}) \tag{15}$$

$$u_m \cong M(y)+m(y)(\tau+\tfrac{1}{2})+\tfrac{1}{2}\mu(y)\{(\tau+\tfrac{1}{2})^2-1/12\} \tag{16}$$

for $0 < y < 1$, $-1 < \tau < 0$, and

$$u_a \cong A(y)+a(y)(\tfrac{1}{2}-\tau) \tag{17}$$

$$u_m \cong N(y)+n(y)(\tfrac{1}{2}-\tau)+\tfrac{1}{2}\nu(y)\{(\tfrac{1}{2}-\tau)^2-1/12\} \tag{18}$$

for $0 < y < 1$, $0 < \tau < 1$.

The equations to be satisfied by the unknown functions G, g, etc., will come from the weak form of the Nusselt equations, such as equation (14). With the choice

$$\chi_1(\tau) = 1$$

four equations are obtained

$$\partial G/\partial y = -\Lambda_g(G-M) \tag{19}$$

$$\partial A/\partial y = \Lambda_a(A-N) \tag{20}$$

$$m = \pi_g(G-M) \tag{21}$$

$$n = -\pi_a(A-N). \tag{22}$$

A further four equations can be obtained with the choice

$$\chi_2(\tau) = \tau$$

$$\partial g/\partial y = -\Lambda_g(g-m) \tag{23}$$

$$\partial a/\partial y = \Lambda_a(a-n) \tag{24}$$

$$\mu = \pi_g(g-m) \tag{25}$$

$$\nu = -\pi_a(a-n) \tag{26}$$

(using equations (19)–(22)). The periodicity equations (10) give

$$m = n \tag{27}$$

and

$$M+\pi_g(g-m)/12 = N-\pi_a(a-n)/12. \tag{28}$$

The boundary conditions become simply

$$G = 1, g = 0 \quad \text{at } y = 0$$

$$A = a = 0 \quad \text{at } y = 1. \tag{29}$$

Equations (19)–(28) form a system of ten equations for the ten unknown functions of y in the approximate solution (equations (15)–(18)), hence with this form of approximate solution it is appropriate to take $J = 2$. It would be possible to introduce further terms into the approximate solution and to take a larger value for J. However, since the above solution will be shown to give a good approximation to all the features of interest of the problem this has not been done.

4. RESULTS OF THE METHOD

Equations (19)–(28) can be regarded as a fourth-order system of differential equations subject to the four boundary conditions (29). As the system is linear, it is appropriate to obtain the solution in terms of superposition of four fundamental solutions which are derived in the Appendix. In the solution the parameter

$$\varepsilon = \Lambda_g\pi_a\pi_g/12(\Lambda_a+\Lambda_g) \tag{30}$$

is regarded as small and all terms in ε^2 are ignored. This is consistent with the assumption of the form of the time variation of u, and it should be a good approximation for π_a and $\pi_g < 2$.

For convenience the sequence $\{G, A, M, N, g, a, m\}$ is denoted by $\mathbf{F}$. Writing the four fundamental solutions as $\mathbf{F}_j$ for $j = 1, 2, 3$ and 4, the general solution of equations (19)–(28) is

$$\mathbf{F} = \sum_{j=1}^{4} b_j\mathbf{F}_j. \tag{31}$$

Use of boundary conditions (29) yields a system of four linear equations for the coefficients b_j. In general the solution of these linear equations is numerical, however, for cases where the dimensionless lengths are sufficiently large for $e^{-\Lambda_g}$ and $e^{-\Lambda_g}$ to be neglected an analytic solution becomes practical. For this latter case the dimensionless outlet temperatures are as given below.

4.1. *Balanced regenerator*

A regenerator is defined to be balanced if $\Lambda_g/\pi_g = \Lambda_a/\pi_a$. In this case the time average of the dimensionless air outlet temperature is $A(0)$, i.e. the second element of $\mathbf{F}$ evaluated at $y = 0$, which is

$$A(0) = (\Lambda_g-2\varepsilon)/(1+k+\Lambda_g-2\varepsilon) \tag{32}$$

where

$$k = \Lambda_g/\Lambda_a. \tag{33}$$

Similarly the time averaged gas outlet temperature is

$$G(1) = (1+k)/(1+k+\Lambda_g-2\varepsilon). \tag{34}$$

It is of interest to note that the limits of these terms as $\varepsilon \to 0$ are the outlet temperatures for a continuously acting recuperator. For details of the analysis of a recuperator see, e.g. Kays and London [10]. The dimensionless air and gas outlet temperature swings can be seen from equations (15) and (17) to be $a(0)$ and $g(1)$, respectively. These take the values

$$a(0) = g(1) = \pi_g(1-\varepsilon/(1+k))/(1+k+\Lambda_g-2\varepsilon). \quad (35)$$

4.2. *Unbalanced regenerator*

The mean air outlet temperature is

$$A(0) = \{(E-1)(\Lambda_g+f)-\varepsilon f(E/\beta+1)\}/\{(E-\beta)(\Lambda_g+f)-\varepsilon f(E/\beta+\beta)\} \quad (36)$$

where

$$f = (\Lambda_a\pi_g - \Lambda_g\pi_a)/(\pi_g+\pi_a) \quad (37)$$

$$E = e^f \quad (38)$$

and

$$\beta = \Lambda_g\pi_a/\Lambda_a\pi_g. \quad (39)$$

The air outlet temperature swing is

$$a(0) = \pi_g f\{1-\varepsilon\beta/(1+k)\}/\{(E-\beta)(\Lambda_g+f)-\varepsilon f(E/\beta+\beta)\}. \quad (40)$$

The mean gas outlet temperature is

$$G(1) = E\{(1-\beta)(\Lambda_g+f)-\varepsilon f(1/\beta-1)\}/\{(E-\beta)(\Lambda_g+f)-\varepsilon f(E/\beta+\beta)\} \quad (41)$$

and the gas outlet temperature swing is

$$g(1) = \pi_g Ef\{\beta-\varepsilon/(1+k)\}/\{(E-\beta)(\Lambda_g+f)-\varepsilon f(E/\beta+\beta)\}. \quad (42)$$

Other qualities of interest, e.g. the variation of metal temperature, can be determined from the detailed solution in the Appendix.

5. REGENERATOR WITH TWO SETS OF HEAT EXCHANGE ELEMENTS

The heat exchange elements for a power station regenerative air heater will ideally provide high thermal performance and a low susceptibility to fouling and corrosion. In a number of air heaters it has been found advantageous to have two completely different designs of element to cope with the varying conditions throughout the heater. Over most of the depth the regenerator is fitted with elements of high thermal performance, but close to the air inlet end, where the lowest temperatures occur, the elements are chosen to have a low susceptibility to fouling, which generally means that the thermal performance is lower. In consequence the values of Λ and π will be different for the two elements.

For high-temperature applications of regenerators the fluid properties vary strongly with temperature, especially if radiative heat transfer becomes significant. One approximation which is used is to split the regenerator into a number of zones and to take the fluid properties appropriate to a representative temperature in each zone. The following is directly applicable to a two-zone approximation for a regenerator, and the generalization to three or more zones is straightforward.

In each of the two regions of the regenerator the temperatures will be approximated by equations of the form of equations (15)–(18). In each region the functions G, g, A, a, etc. will satisfy equations of the form of equations (19)–(28), and the fundamental solutions will be of the form given in the Appendix with the constants Λ_g, Λ_a, π_g and π_a taking the appropriate values in each region. The solution will take the form

$$\mathbf{F} = \sum_{j=1}^{4} b_j\mathbf{F}_j \quad 0 < y < z \quad (43)$$

and

$$\mathbf{F} = \sum_{j=1}^{4} b_{4+j}\mathbf{F}_j \quad z < y < 1 \quad (44)$$

where the interface between the two regions is $y = z$.

At the interface between the two elements or zones it is clear that the gas and air temperatures must be continuous at all times, so that G, g, A and a are continuous functions of y at $y = z$. It is not appropriate to impose any condition of continuity on the solid temperature as axial heat conduction along the solid has been ignored. In practice in a rotary regenerator with two sets of elements, the upper and lower elements are separated by a small gap, so that even if axial heat conduction was included in the model, conditions of continuity of the element temperature at the interface would not be imposed.

Use of the four boundary conditions (29) and the four interface conditions

$$G, g, A, a, \quad \text{continuous at } y = z \quad (45)$$

give a system of eight linear equations from which the eight constants b_j in equations (43) and (44) may be determined. In this case an analytic solution of the equations is not practical, and the equations are solved numerically.

6. DISCUSSION

One can investigate the effect of the approximations made in the solution of equations (1) and (2) by comparing the solution with numerical results from a finite difference scheme. The finite difference method used was developed by Donovan (private communication) and is similar in form to that described by Willmott [4]. There are two approximations made in obtaining an analytic solution to the problem, which are that the time variation of the actual temperature distribution should be similar to the assumed form (equations (15)–(18)) and that the parameter ε is small compared with unity. Both of these are reasonable approximations provided the reduced periods π_a and π_g are not too large and that π_a and π_g

Table 1. Comparison of methods for a two-element case (all temperatures dimensionless)

	Heat balance method	Finite difference 80 × 80	Finite difference 40 × 40	Hausen
Gas inlet end				
Mean air temperature	0.7608	0.7616	0.7604	0.7647
Metal temperatures:				
mean	0.8925	0.8918	0.8902	0.8945
hottest	0.9494	0.9482	0.9463	0.9608
coldest	0.8212	0.8208	0.8188	0.8282
$x = 0.65l$				
Mean gas temperature	0.5310	0.5314	0.5294	
Mean air temperature	0.1871	0.1886	0.1859	
Metal temperatures:				
mean	0.3765	0.3776	0.3753	
hottest	0.4749	0.4757	0.4737	
coldest	0.2890	0.2890	0.2863	
$x = 0.7l$				
Mean gas temperature	0.5035	0.5031	0.5043	
Mean air temperature	0.1533	0.1533	0.1537	
Metal temperatures:				
mean	0.3533	0.3533	0.3537	
hottest	0.3769	0.3769	0.3773	
coldest	0.3333	0.3337	0.3333	
Air inlet end				
Mean gas temperature	0.3780	0.3776	0.3784	0.3733
Metal temperatures:				
mean	0.2157	0.2161	0.2169	0.2133
hottest	0.2404	0.2404	0.2412	0.2361
coldest	0.1937	0.1941	0.1945	0.1906

Parameters:

$\Lambda_{g1} = 5.739$, $\Lambda_{a1} = 5.702$, $\Lambda_{g2} = 2.673$, $\Lambda_{a2} = 2.454$

$\pi_{g1} = 1.267$, $\pi_{a1} = 1.029$, $\pi_{g2} = 0.288$, $\pi_{a2} = 0.216$

$\varepsilon_1 = 0.053$, $\varepsilon_2 = 0.003$

Change in element characteristics at $x = 0.666l$

are no larger than 2 and no larger than Λ_a or Λ_g, respectively.

In Tables 1 and 2 the results obtained by the present heat balance method are compared with finite difference results using data from two power station regenerative air heaters. The heater represented in Table 1 has hot and cold elements with different characteristics, and its dimensionless lengths and periods are typical of those from air heaters in operation in the CEGB. Table 2 represents an air heater with a combination of high-performance elements and a slow speed of rotation, which result in larger values of π.

In the tables data is given from the finite difference method using two values for the step lengths of the space and time variables. The truncation error can be estimated by comparing the results with two different step lengths. It is seen that the heat balance method differs from the fine mesh finite difference results by up to 2×10^{-3}, which is slightly less than the estimate of truncation error. For the data in Table 2, which is slightly outside the range of π for which the method is expected to be accurate, the difference between the heat balance method and the finite difference results is around 10^{-2}, which is about twice the estimate of truncation error.

The Hausen method referred to earlier reformulates the regenerator as a recuperator with a slightly modified length. Strictly speaking the method only applies to a heat exchanger fitted with a single element type, although in principle it could be modified for the two-element case. An alternative approach to apply the Hausen method to a heater with two element types is to use the standard Hausen correction term, but with suitably defined overall values for the reduced length and period. This approach was used to obtain the data in the final column of Tables 1 and 2. Hausen's method only gives a direct estimate of the time averaged fluid and metal temperatures. However, it is possible to use these to estimate the heat transfer from the fluids to the metal and hence to estimate the time variation of metal temperature. This is done in the final column of Tables 1 and 2. Note that the estimate of the time averaged metal and fluid temperatures obtained from the Hausen method are slightly in error, whereas the estimates of the time variation of the metal temperatures are noticeably worse.

Table 2. Comparison of methods for an extreme two-element case (all temperatures dimensionless)

	Heat balance method	Finite difference 80 × 80	Finite difference 40 × 40	Hausen
Gas inlet end				
Air temperature:				
mean	0.8888	0.8895	0.8881	0.8926
hottest	0.9554	0.9505	0.9474	
coldest	0.8233	0.8177	0.8167	
Metal temperatures:				
mean	0.9488	0.9484	0.9467	0.9512
hottest	0.9899	0.9902	0.9888	1.0136
coldest	0.8871	0.8801	0.8791	0.8888
$x = 0.5l$				
Gas temperature:				
mean	0.7152	0.7163	0.7145	
hottest	0.8149	0.8079	0.8066	
coldest	0.6155	0.6107	0.6075	
Air temperature:				
mean	0.5249	0.5270	0.5249	
hottest	0.6480	0.6431	0.6375	
coldest	0.4019	0.4123	0.4106	
Metal temperatures:				
mean	0.6288	0.6267	0.6284	0.6267
hottest	0.7365	0.7372	0.7358	0.7365
coldest	0.5120	0.5155	0.5131	0.5204
Air inlet end				
Gas temperature:				
mean	0.3043	0.3036	0.3039	0.3032
hottest	0.3876	0.3911	0.3900	
coldest	0.2210	0.2220	0.2252	
Metal temperature:				
mean	0.1680	0.1687	0.1694	0.1677
hottest	0.2259	0.2262	0.2266	0.2192
coldest	0.1199	0.1206	0.1216	0.1161

Parameters:

$\Lambda_{g1} = 8.947, \quad \Lambda_{a1} = 9.511, \quad \Lambda_{g2} = 5.569, \quad \Lambda_{a2} = 5.764$

$\pi_{g1} = 2.632, \quad \pi_{a1} = 2.190, \quad \pi_{g2} = 0.784, \quad \pi_{a2} = 0.635$

$\varepsilon_1 = 0.233, \quad \varepsilon_2 = 0.020$

Change in element characteristics at $x = 0.84l$

In Table 3 the method is used to estimate the effectiveness, i.e. the dimensionless air outlet temperature of balanced symmetric regenerators with a single element type with dimensionless length Λ in the range 1–10 and dimensionless periods π of 1, 2 and 3. The results are compared with results presented in Schmidt and Willmott [3], computed by the method of Willmott [4]. It is seen that there is good agreement for $\pi = 1$ and 2 apart from the case $\Lambda = 1, \pi = 2$. For $\pi = 3$ the agreement is good for larger values of Λ but becomes poor for $\Lambda = 3$ or less. This is in line with the limitations of the method, since the assumed form of the temperature distribution will not be a good approximation if $\Lambda < \pi$, and the parameter ε cannot be regarded as small if $\pi > 2$.

In Tables 4 and 5 the method is used to predict the dimensionless metal and gas temperatures at the start and end of the gas cycle for long balanced symmetric regenerators. These cases have also been considered by Hill and Willmott [7]. Their results are not presented here, however the maximum difference between the temperatures predicted by the two methods is 10^{-4}.

Finally in Table 6, some cases considered by Nahavandi and Weinstein [6] are presented. The predicted effectiveness (i.e. the dimensionless air outlet temperature) is compared with the generally accepted value for the effectiveness as given for example by Hill and Willmott [7]. It is seen that only the first four cases presented here satisfy the constraints of the method, i.e. $\Lambda \geqslant \pi$ and $\pi \leqslant 2$. For these four cases the agreement is good. For the remainder of the cases considered by Nahavandi and Weinstein the heat balance method gives poor or even nonsensical results with the predicted effectiveness sometimes outside the range [0, 1]. These latter cases, of which four are presented in Table 6, are intended as a demonstration that the heat balance method should not be used outside the parameter range for which it was developed.

The CPU time used to generate all the heat balance

Table 3. Values of the effectiveness for balanced symmetric regenerators as computed by the heat balance method. Values from Schmidt and Willmott [3] are given in parentheses for comparison

Reduced length, Λ	Reduced period, π 1	2	3
1	0.3215 (0.3221)	0.2846 (0.2930)	0.2177 (0.2559)
2	0.4909 (0.4912)	0.4616 (0.4664)	0.4061 (0.4305)
3	0.5936 (0.5937)	0.5731 (0.5757)	0.5342 (0.5477)
4	0.6621 (0.6622)	0.6475 (0.6490)	0.6202 (0.6282)
5	0.7109 (0.7109)	0.7001 (0.7012)	0.6803 (0.6856)
6	0.7474 (0.7474)	0.7392 (0.7400)	0.7242 (0.7280)
7	0.7757 (0.7758)	0.7692 (0.7699)	0.7576 (0.7605)
8	0.7983 (0.7984)	0.7931 (0.7936)	0.7838 (0.7861)
9	0.8168 (0.8169)	0.8125 (0.8129)	0.8049 (0.8068)
10	0.8322 (0.8322)	0.8286 (0.8289)	0.8222 (0.8238)

results used in the tables was 0.11 s on an Amdahl 5870 mainframe computer or 25 s on an Olivetti M24 personal computer. For comparison an open finite difference method similar to the Willmott method [4] used 20 gridpoints and 20 time steps for each period for a typical case, and took 23 cycles to reach equilibrium. The corresponding CPU times were 0.47 s on the Amdahl and 115 s on the Olivetti for this one case.

7. CONCLUSIONS

The steady-state temperature distribution in a regenerator has been determined by an approximate analytic method based on the heat balance method. The solution has been shown to be a good approximation to the temperature distribution in rotary regenerators in power station boilers. The method is restricted to cases where the reduced periods are not too large, however, there is no restriction on the reduced lengths. A novel feature of the method is that it can readily cope with a number of regenerators in series. This has applications to power station air heaters where different heat exchange elements are usually fitted in the hot and cold zones of the heater. Another

Table 4. Values of the dimensionless metal temperature at the start and end of the gas cycle for long balanced symmetric regenerators with $\pi = 0.1$

y	Dimensionless metal temperature at start of gas cycle Λ = 100	Λ = 250	Λ = 500	Dimensionless metal temperature at end of gas cycle Λ = 100	Λ = 250	Λ = 500
0	0.9897	0.9958	0.9974	0.9907	0.9962	0.9981
0.1	0.8917	0.8966	0.8983	0.8927	0.8970	0.8985
0.2	0.7936	0.7974	0.7987	0.7946	0.7978	0.7989
0.3	0.6956	0.6982	0.6991	0.6966	0.6986	0.6993
0.4	0.5976	0.5990	0.5995	0.5985	0.5994	0.5997
0.5	0.4995	0.4998	0.4999	0.5005	0.5002	0.5001
0.6	0.4015	0.4006	0.4003	0.4025	0.4010	0.4005
0.7	0.3034	0.3014	0.3007	0.3044	0.3018	0.3009
0.8	0.2054	0.2022	0.2011	0.2064	0.2026	0.2013
0.9	0.1074	0.1030	0.1015	0.1083	0.1034	0.1017
1.0	0.0093	0.0038	0.0019	0.0103	0.0042	0.0021

Table 5. Values of the dimensionless gas temperature at the start and end of the gas cycle for long balanced symmetric regenerators with $\pi = 0.1$

y	Dimensionless gas temperature at start of gas cycle Λ = 100	Λ = 250	Λ = 500	Dimensionless gas temperature at end of gas cycle Λ = 100	Λ = 250	Λ = 500
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9015	0.9006	0.9003	0.9025	0.9010	0.9005
0.2	0.8034	0.8014	0.8007	0.8044	0.8018	0.8009
0.3	0.7054	0.7022	0.7011	0.7064	0.7026	0.7013
0.4	0.6074	0.6030	0.6015	0.6083	0.6034	0.6017
0.5	0.5093	0.5038	0.5019	0.5103	0.5042	0.5021
0.6	0.4113	0.4047	0.4023	0.4123	0.4050	0.4025
0.7	0.3132	0.3054	0.3027	0.3142	0.3058	0.3029
0.8	0.2152	0.2062	0.2031	0.2162	0.2066	0.2033
0.9	0.1172	0.1069	0.1035	0.1181	0.1073	0.1037
1.0	0.0191	0.0077	0.0039	0.0201	0.0081	0.0041

Table 6. Values of the effectiveness for unbalanced asymmetric regenerators (only the first four cases are within the range of validity of the heat balance method)

Λ_g	Λ_a	π_g	π_a	Effectiveness (heat balance)	Effectiveness (Hill and Willmott)
1.4	2.0	1.0	1.0	0.5266	0.5271
2.8	4.0	2.0	2.0	0.7062	0.7087
1.0	2.0	1.0	1.0	0.5510	0.5516
2.0	4.0	2.0	2.0	0.7425	0.7462
4.0	8.0	4.0	4.0	0.8761	0.8929
8.0	16.0	8.0	8.0	1.0730	0.9717
0.2	2.0	2.0	2.0	0.5233	0.5535
0.8	8.0	8.0	8.0	1.0110	0.8010

application is to high-temperature regenerators where it is convenient to approximate the temperature variation of the fluid properties by splitting the regenerator into a number of zones, in each of which the fluid properties are assumed to be constant.

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REFERENCES

1. H. Hausen, Improved calculations for heat transfer in regenerators, *Verfahrenstecknik Z. Ver. Dt. Ing.* **2,** 31–43 (1942).
2. R. J. McDuff and N. D. Clark, Ljungstrom air preheater design and operation, 2: corrosion and fouling, *Combustion* **47,** 24–30 (1976).
3. F. W. Schmidt and A. J. Willmott, *Thermal Energy Storage and Regeneration*. McGraw-Hill, New York (1981).
4. A. J. Willmott, Digital computer simulation of a thermal regenerator, *Int. J. Heat Mass Transfer* **7,** 1291–1303 (1964).
5. C. E. Iliffe, Thermal analysis of counterflow regenerative heat exchanger, *J. Instn Mech. Engrs* **159,** 363–372 (1948).
6. A. N. Nahavandi and A. S. Weinstein, A solution to the periodic flow regenerative heat exchanger problem, *Appl. Scient. Res.* **A10,** 335–348 (1961).
7. A. Hill and A. J. Willmott, A robust method for regenerative heat exchange calculations, *Int. J. Heat Mass Transfer* **30,** 241–249 (1987).
8. D. Langford, The heat balance integral method, *Int. J. Heat Mass Transfer* **16,** 2424–2428 (1973).
9. B. Noble, *Applications of Undergraduate Mathematics in Engineering*. MacMillan, London (1971).
10. W. M. Kays and A. L. London, *Compact Heat Exchangers*. McGraw-Hill, New York (1964).

APPENDIX

Equations (19)–(28) form a fourth-order system of differential equations which are subject to the four boundary conditions (29). For convenience a solution of the system is denoted by the sequence

$$\mathbf{F} = \{G, A, M, N, g, a, m\}. \quad \text{(A1)}$$

Solutions to the system can be obtained by assuming that $\mathbf{F}$ has the form $\mathbf{C}e^{\alpha y}$. Equations (19)–(27) are satisfied by

$$\mathbf{F} = \{1, 1, 1, 1, 0, 0, 0\} \quad \text{(A2)}$$

which corresponds to $\alpha = 0$, and by

$$\mathbf{F} = \{\Lambda_g/\pi_g, \Lambda_a/\pi_a, (\Lambda_g+\alpha)/\pi_g, (\Lambda_a-\alpha)/\pi_a, -\Lambda_g\alpha/(\Lambda_g+\alpha), -\Lambda_a\alpha/(\Lambda_a-\alpha), -\alpha\}\, e^{\alpha y} \quad \text{(A3)}$$

for any value of α, while equation (28) gives

$$\{\Lambda_a\pi_g - \Lambda_g\pi_a - (\pi_g+\pi_a)\alpha\} \times \{(\Lambda_a-\alpha)(\Lambda_g+\alpha) - \alpha^2\pi_a\pi_g/12\} = 0 \quad \text{(A4)}$$

which is satisfied by $\alpha = f$, where

$$f = (\Lambda_a\pi_g - \Lambda_g\pi_a)/(\pi_g+\pi_a) \quad \text{(A5)}$$

or by

$$(\Lambda_a-\alpha)(\Lambda_g+\alpha) = \alpha^2\pi_a\pi_g/12. \quad \text{(A6)}$$

One can now set

$$\varepsilon = \Lambda_g\pi_g\pi_a/12(\Lambda_g+\Lambda_a) \quad \text{(A7)}$$

and restrict attention to problems where ε is small and ignore terms in ε^2. Note that this is consistent with the assumption of the time variation of u, and it should be a good approximation for π_a and $\pi_g < 2$. Equation (A6) is approximately satisfied by

$$\alpha = \Lambda_g + \Lambda_g\varepsilon \quad \text{(A8)}$$

and

$$\alpha = \Lambda_a - \Lambda_a\varepsilon/k \quad \text{(A9)}$$

where

$$k = \Lambda_g/\Lambda_a. \quad \text{(A10)}$$

It is now convenient to consider separately the cases of a balanced regenerator, i.e. a regenerator for which $f = 0$, and an unbalanced regenerator.

Balanced regenerator

For this case there is a repeated root $\alpha = 0$ and instead one has a solution which is linear in y. Two fundamental solutions are then

$$\mathbf{F}_1 = \{1, 1, 1, 1, 0, 0, 0\} \quad \text{(A11)}$$

$$\mathbf{F}_2 = \{1+k-\Lambda_g y, -\Lambda_g y, k-\Lambda_g y, k-\Lambda_g y, \pi_g, \pi_g, \pi_g\}. \quad \text{(A12)}$$

Substituting equations (A8) and (A9) into equation (A2) and retaining only the highest term in ε throughout one obtains two other fundamental solutions as

$$\mathbf{F}_3 = \{\varepsilon, \varepsilon, 0, \varepsilon(1+k), \pi_g, \varepsilon\pi_g/(1+k), \varepsilon\pi_g\}\, e^{-\Lambda_g} \quad \text{(A13)}$$

$$\mathbf{F}_4 = \{-\varepsilon, -\varepsilon, -\varepsilon(1+k)/k, 0, \varepsilon\pi_g/(1+k), \pi_g, \varepsilon\pi_g/k\}\, e^{\Lambda_a(y-1)}. \quad \text{(A14)}$$

The general solution of equations (19)–(28) is given by

$$\mathbf{F} = \sum_{i=1}^{4} b_i\mathbf{F}_i \quad \text{(A15)}$$

and boundary conditions (29) give a system of four linear equations from which the constants b_i can be determined. In general a numerical evaluation of the constants b_i is appropriate, however if the dimensionless lengths Λ_g and Λ_a are large enough for the terms $\exp(-\Lambda_a)$ and $\exp(-\Lambda_g)$ to be neglected, the equations simplify considerably. In this case the solution is

$$\mathbf{F} = \{(\Lambda_g-\varepsilon)\mathbf{F}_1 + \mathbf{F}_2 - \mathbf{F}_3 - \mathbf{F}_4\}/(1+k+\Lambda_g-2\varepsilon). \quad \text{(A16)}$$

The time outlet temperatures and temperature swings, equations (32), (34) and (35), follow immediately by substitution.

Unbalanced regenerator

For an unbalanced regenerator, the quantity f given by equation (A4) is nonzero. The following notation is introduced:

$$E = e^f \quad \text{and} \quad \beta = \Lambda_g\pi_a/\Lambda_a\pi_g. \quad \text{(A17)}$$

Four fundamental solutions to equations (19)–(28) are

$$\mathbf{F}_1 = \{1, 1, 1, 1, 0, 0, 0\} \quad \text{(A18)}$$

$$\mathbf{F}_2 = \{\beta, 1, \pi_a(\Lambda_a+\Lambda_g)/\Lambda_a(\pi_a+\pi_g), \pi_a(\Lambda_a+\Lambda_g)/\Lambda_a(\pi_a+\pi_g), -\beta\pi_g f/(\Lambda_g+f), -\pi_g f/(\Lambda_g+f), -\pi_a f/\Lambda_a\} \quad \text{(A19)}$$

$$\mathbf{F}_3 = \{\varepsilon, \varepsilon/\beta, 0, \varepsilon(1+k)/\beta, \pi_g, \varepsilon\pi_g/(1+k), \varepsilon\pi_g\}\, e^{-\Lambda_g y}. \quad \text{(A20)}$$

$$\mathbf{F}_4 = \{-\varepsilon, -\varepsilon/\beta, -\varepsilon(1+k)/k, 0, \varepsilon\pi_g/(1+k), \pi_g, \varepsilon\pi_g/k\}\, e^{\Lambda_a(y-1)}. \quad \text{(A21)}$$

As before the general solution is given by equation (A15), and a numerical solution of the resulting linear equations is required, except for the case where Λ_a and Λ_g are large enough for the exponential terms including them to be neglected. In this case the solution is

$$\mathbf{F} = \{E(\Lambda_g+f-\varepsilon f/\beta)\mathbf{F}_1-(\Lambda_g+f)\mathbf{F}_2-\beta f\mathbf{F}_3-Ef\mathbf{F}_4\}/\{(E-\beta)(\Lambda_g+f)-\varepsilon f(\beta+E/\beta)\}. \quad \text{(A22)}$$

Once again the outlet temperatures and temperature swings, equations (36) and (40)–(42), follow by substitution.

ANALYSE THERMIQUE APPROCHEE D'UN ECHANGEUR DE CHALEUR REGENERATEUR

Résumé—On décrit une analyse thermique approchée d'un régénérateur thermique. L'approximation repose sur le fait que les paramètres adimensionnels, tels que les périodes réduites, ne sont pas grands, condition qui est précisée et qui est réalisée pour tous les réchauffeurs d'air. La méthode donne les températures moyennes dans le temps à la sortie, qui sont équivalentes à celle déduites d'une analyse de Hausen (*Verfahrenstecknik Z. Ver. Dt. Ing.* **2**, 31–43 (1942)). La méthode prédit aussi la variation des températures du fluide et des éléments de l'échangeur avec la position et le temps. La méthode est généralisée pour couvrir le cas d'un régénérateur avec deux zones ou plus ayant des coefficients de transfert thermique différents. Cela s'applique à des réchauffeurs d'air où des éléments d'échangeur différents sont fréquemment utilisés dans les zones chaudes et froides du réchauffeur, et à des régénérateurs à haute température où la variation des propriétés du fluide avec la température peut être approchée en séparant le régénérateur en plusieurs zones dans chacune desquelles les propriétés sont constantes. Les résultats sont comparés avec la solution par différences finies. L'aptitude de la méthode est démontrée pour des régénérateurs longs et les résultats sont comparés avec la solution de Hausen et avec les résultats classiques donnés dans les publications.

EINE THERMISCHE NÄHERUNGSANALYSE FÜR EINEN REGENERATIVWÄRMETAUSCHER

Zusammenfassung—Der Aufsatz beschreibt eine Näherungsanalyse für einen regenerativen Wärmetauscher. Das Näherungsverfahren beruht auf der Tatsache, daß die dimensionslosen Parameter, auch als reduzierte Periodendauer bekannt, nicht zu große Werte annehmen. Diese für alle Kraftwerks-Lufterhitzer zutreffende Bedingung wird in dem Aufsatz näher erläutert. Für die Größe der dimensionslosen Längen gibt es keine Einschränkung. Das Verfahren liefert zeitlich gemittelte Austrittstemperaturen, welche mit den von Hausen (*Verfahrenstecknik Z. Ver. Dt. Ing.* **2**, 31–43 (1942)) bei der Analyse eines äquivalenten Rekuperators gefundenen Werten übereinstimmen. Weiterhin wird die Änderung der Fluidtemperatur und der Temperatur der Wärmeübertragungselemente in Abhängigkeit von Zeit und Ort bestimmt. Das Verfahren gilt für den allgemeinen Fall eines Regenerators mit zwei oder mehr Abschnitten, von denen jeder unterschiedliche Wärmeübergangskoeffizienten haben kann. Dieser Fall kommt in regenerativen Kraftwerks-Lufterhitzern vor, bei denen unterschiedliche Wärmeübertragungselemente wechselweise in heißen und kalten Bereichen des Wärmetauschers verwendet werden. Eine weitere Anwendungsmöglichkeit sind Hochtemperatur-Regeneratoren, bei denen zur Annäherung der Änderung der Stoffeigenschaften des Fluids mit der Temperatur eine Aufteilung des Regenerators in mehrere Bereiche erfolgt, in denen diese Größen als konstant angesehen werden. Die Ergebnisse werden mit denen einer Lösung nach dem Finite-Differenzen-Verfahren für Betriebsdatensätze verglichen. Weiterhin wird die Eignung des Verfahrens für lange Regeneratoren gezeigt und die Ergebnisse mit der Lösung von Hausen und Literaturergebnissen verglichen.

ПРИБЛИЖЕННЫЙ ТЕПЛОВОЙ АНАЛИЗ РЕГЕНЕРАТИВНОГО ТЕПЛООБМЕННИКА

Аннотация—Представлен приближенный тепловой анализ регенеративного теплообменника. Используемая аппроксимация основана на том, что безразмерные параметры, известные как приведенные периоды, не слишком велики—условие, которое уточняется в настоящей работе и которое справедливо для всех воздухонагревателей электростанций. Не существует верхнего предела уменьшенных размеров, к которому данный метод может быть применен. Получены осредненные по времени температуры жидкости на выходе, эквивалентные температурам, выведенным в результате анализа аналогичного рекуператора Хаузена (*Verfahrenstecknik Z. Ver. Dt. Ing.* **2**, 31–43 (1942)). Рассчитано временное и пространственное изменение температур жидкости и теплообменного элемента. Метод обобщен на случай регенератора с двумя или более зонами, имеющими коэффициенты теплообмена, отличающиеся для каждой зоны. Метод применим в регенеративных воздухонагревателях электростанций, где различные теплообменные элементы часто используются в горячих и холодных зонах нагревателя, и в высокотемпературных регенераторах, где изменение свойств жидкости с температурой может быть аппроксимировано путем разделения регенератора на несколько зон с постоянными свойствами в каждой из них. Результаты сравниваются с решением в конечных разностях задачи регенератора для набора данных для станции. Продемонстрирована пригодность метода для длинных регенераторов; результаты сравниваются с решением Хаузена и стандартными данными, имеющимися в литературе.